Principle of Indifference & Imprecise Credences

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The Principle of Indifference

The Principle of Indifference provides a constraint on your assignment of prior probabilities.

This principle is also sometimes called *The Principle of Insufficient Reason*.

POI If you have no more reason to think P than Q, and you have no more reason to think Q than P, then $Cr(P) = Cr(Q)$.

The Partition Problem. The Principle of Indifference is problematic because what advice it offers depends on how the possibilities are partitioned.

EXAMPLE: THE LIGHT SWITCH & THE URN. There is an urn that contains red, green, and blue marbles. A marble is chosen at random. The light is turned on if and only if a red marble is chosen. What's your credence that the light is on?

Here are two possible answers:

1. $Cr(\text{Light is On}) = \frac{1}{2}$.
   The light is either on or off. You have no more reason to think that it is on than off (and *vice versa*). So, by the Principle of Indifference, you should assign credence $\frac{1}{2}$ to both.
2. $Cr(\text{Light is On}) = \frac{1}{3}$.
   The light is on if and only if a red marble has been selected. There are three possible colors. And you have no more reason to think that the selected marble is one of those colors than any other. So, by the Principle of Indifference, you should assign $\frac{1}{3}$ to each of these possibilities; and, thus, $\frac{1}{3}$ to light being on.

Huemer offers a suggestion for picking the "privileged" partition to which the Principle of Indifference should be applied.

EPP Apply the Principle of Indifference to the partition that is most explanatorily basic.

Problem: Can't there be multiple partitions, none of which more explanatorily basic than the others?

EXAMPLE: THE MYSTERY CUBE FACTORY. You know that the factory produces cubes of some particular size. You know the length of the cubes produced is somewhere between 0 and 2 feet long.

	Length $= \ell$		Area $= a$
L_1	$0 \leq \ell \leq 1$	A_1	$0 \leq a \leq 1$
L_2	$1 \leq \ell \leq 2$	A_2	$1 \leq a \leq 2$
		A_3	$2 \leq a \leq 3$
		A_4	$3 \leq a \leq 4$

L_1 is logically equivalent to A_1.
L_2 is logically equivalent to $(A_2 \vee A_3 \vee A_4)$.

Roger White's Response: The problem isn't with **POI.** We can generate an absurd result without invoking the Principle of Indifference.

AN ARGUMENT TO AN ABSURD RESULT

1.	$L_1 \approx L_2$	[Assumption]
2.	$A_1 \approx A_2 \approx A_3 \approx A_4$	[Assumption]
3.	$L_1 \approx A_1$	[Logical Equivalence]
4.	$L_2 \approx (A_2 \vee A_3 \vee A_4)$	[Logical Equivalence]
5.	$L_1 \approx (A_2 \vee A_3 \vee A_4)$	[1, 4, Transitivity]
6.	$A_1 \approx (A_2 \vee A_3 \vee A_4)$	[3, 5, Transitivity]
7.	$A_2 \approx (A_2 \vee A_3 \vee A_4)$	[2, 6, Transitivity]
	?!!?	

The argument relies on two principles:

Logical Equivalence. If P and Q are logically equivalent, then $P \approx Q$.

Transitivity. If $P \approx Q$, and $Q \approx R$, then $P \approx R$.

Reasons to Like POI: (1) Argument from Cases, (2) Argument from Statistical Inference, (3) Evidentialist Argument*

* ... evidential symmetry demands symmetry of opinion — but why think that one's opinion must always be represented by a single standard probability function?

Imprecise Credences

1. **Gradation.** Your epistemic state is *a set* of credence functions $\mathcal{C}$.
2. **Probabilism.** Each credence function in $\mathcal{C}$ is a probability function.
3. **Conditionalization.** You learn E by conditionalizing each of the credence functions in $\mathcal{C}$ on E.
$$\mathcal{C}^+(X) = \{Cr(X \mid E) \mid \forall Cr \in \mathcal{C}\}$$
4. **Supervaluationism.** If f is a determinate feature of your epistemic state, then f corresponds to a property had by *every* credence function in your representor.

Call $\mathcal{C}$ your *representor.*

Example: If you are determinately more confident in P than Q, then for every $Cr \in \mathcal{C}$, $Cr(P) > Cr(Q)$.

Imprecise credences offer us, perhaps, a better way of representing evidential symmetry than **POI.** But there are problems: *dilation.*

Learning E *dilates* your opinion about X, when $\mathcal{C}(X) = [x, y]$, and, after learning E, $\mathcal{C}^+(X) = [x - \epsilon, y + \delta]$, where $\epsilon, \delta > 0$.

EXAMPLE: WHITE'S MYSTERY COIN. You haven't a clue as to whether P. But you know that I know whether P. I agree to write "P" on one side of a fair coin, and "$\neg P$" on the other, with *whichever one is true going on the heads side.* We toss the coin and observe that it happens to land on "P".

(a) You haven't a clue about P, so $\mathcal{C}(P) = [0,1]$. **(b)** The coin is fair, so $\mathcal{C}(Heads) = \frac{1}{2}$. **(c)** When you see that the coin landed "P", you learn *Heads if and only if P*, so $\mathcal{C}^+(P) = \mathcal{C}^+(Heads)$. **(d)** $\mathcal{C}^+(P) = \mathcal{C}(P)$. So, $\mathcal{C}^+(Heads) = [0,1]$.

Proof of (d). Let Cr be an arbitrary function in $\mathcal{C}$.

$$\begin{aligned} Cr(P \mid H \equiv P) &= \frac{Cr(P \wedge (H \equiv P))}{Cr(H \equiv P)} \\ &= \frac{Cr(H \wedge P)}{Cr(H \wedge P) + Cr(\neg H \wedge \neg P)} \\ &= \frac{Cr(H) \cdot Cr(P)}{Cr(H) \cdot Cr(P) + Cr(\neg H) \cdot Cr(\neg P)} \\ &= \frac{Cr(P)}{Cr(P) + Cr(\neg P)} = Cr(P) \end{aligned}$$

Problem: Reflection Violation. You know the coin will either land "P" or "$\neg P$". If it lands "P", $\mathcal{C}^+(Heads) = [0,1]$. If it lands "$\neg P$", $\mathcal{C}^+(Heads) = [0,1]$. But, $\mathcal{C}(Heads) = \frac{1}{2}$. So, you are in violation of **The Reflection Principle:**

$$Cr\left(X \mid Cr^+(X) = x\right) = x$$

$\mathcal{C}\left(H \mid \mathcal{C}^+(H) = [0,1]\right) \neq [0,1]$. Is this a problem for the Imprecise Credence View?

The proof relies on the fact that, for every $Cr \in \mathcal{C}$, $Cr(H) = \frac{1}{2}$, and that H and P are *statistically independent:* $Cr(H \wedge P) = Cr(H) \cdot Cr(P)$.