Marx's Theory of Exploitation

Notes from Robert Paul Wolff's *"Marx's Labor Theory of Value"*.

A Theory of Value (in Smith, Ricardo, and Marx)

What determines something's *price*?

- *Natural Price:* "the central price(s), to which the prices of all commodities are continually gravitating." [Smith, I.7]
- *Market Price:* the prices at which, on particular occasions, commodities actually exchange.

The Three Central Issues of Classical Political Economy:

1. The nature of *real wealth.*
2. The causes of economic *growth.*
3. The determinants of the *distribution* of real wealth.

In order to handle these issues, we need a theory of prices.

Adam Smith: If little stock and land are owned, commodities will exchange in proportion to the quantity of *labor* required for their production.

David Ricardo: (1) Demonstrates that rent plays no role in the determination of price (2) Argues that the exchange of commodities is regulated by the quantity of labor *directly and indirectly* required for their production.

According to Ricardo, rent is a portion of profits appropriated by landlords at the expense of capitalists.

RICARDO MODEL

Inputs						**Outputs**
L_A	A_A	B_A	C_A	...	yields:	O_A
L_B	A_B	B_B	C_B	...	yields:	O_B
L_C	A_C	B_C	C_C	...	yields:	O_C
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
L_n	A_n	B_n	C_n	...	yields:	O_n

Suppose the economy consists of n one-product industries with one technique of production for each commodity, and a single quality of labor. Assume (for simplicity) that all commodities are themselves directly or indirectly required for the production of all other commodities.

I_J = the amount of commodity I used in producing commodity J.

O_I = the output of commodity I in the industry producing commodity I.

L_I = the quantity of labor directly required in the production of commodity I.

Assume that $O_I \geq (I_A + I_B + \cdots + I_n)$, for all I. Otherwise, the economy is not sustainable.

Unit Input Coefficients: Divide each industry by the magnitude of its output to derive the quantities of inputs required for one unit of output. (We can write a_{ij} to represent the amount of commodity j required to produce one unit of commodity i.)

Labor Values: Let $\lambda_1, \lambda_2, \ldots, \lambda_n$ be variables that represent the amount of labor directly or indirectly required to produce one unit of a commodity. (So, λ_i is the total amount of labor directly or indirectly required to produce one unit of commodity i.)

$$
\begin{aligned}
l_1 + a_{11}\lambda_1 + a_{12}\lambda_2 + a_{13}\lambda_3 + \cdots + a_{1n}\lambda_n &= \lambda_1 \\
l_2 + a_{21}\lambda_1 + a_{22}\lambda_2 + a_{23}\lambda_3 + \cdots + a_{2n}\lambda_n &= \lambda_2 \\
&\vdots \\
l_n + a_{n1}\lambda_1 + a_{n2}\lambda_2 + a_{n3}\lambda_3 + \cdots + a_{nn}\lambda_n &= \lambda_n
\end{aligned}
$$

We have a system of equations, which we can represent as a matrix. Let l be a column vector of direct labor inputs; let A be the $n \times n$ matrix of unit input coefficients; and λ is the column vector of labor value variables.

The system of equations can be represented as

$$l + A\lambda = \lambda \tag{1}$$

Assuming that each commodity is directly or indirectly required for the production of all others, we can solve for λ :

$$\lambda = (1 - A)^{-1}\lambda \tag{2}$$

Ricardo's Claim: The *natural price* of a commodity is proportional to the quantities of labor directly or indirectly required in their production.

In order to *prove* this, we must solve a system of price & profit equations and demonstrate that the price-vector solution is (i) unique, (ii) positive, and (iii) proportional to λ (the vector of labor values).

w is money wage paid to laborers

π is the uniform rate of return on capital investment

p is the vector of prices

$$(lw + Ap)(1 + \pi) = p \tag{3}$$

(The system has n equations with $(n + 2)$ unknowns: n prices, w, and π. Select one price arbitrarily "as numeraire" and set it equal to 1, converting the other prices into *relative prices.* The system can be made solvable only by specifying the *wage:* assume a fixed real wage (a fixed market basket of goods per unit of labor) represented by a $(n \times 1)$ column vector b of goods per unit of labor).

$$w = bp \tag{4}$$

$$(lbp + Ap)(1 + \pi) = p \tag{5}$$

l is an $(n \times 1)$ vector.

b is a $(1 \times n)$ vector.

So, lb is an $(n \times n)$ matrix.

It represents the non-labor inputs, per unit of commodity output, required *for the labor inputs* into the n industries.

Factor out p and call $A^\star = lb + A$ the *Augmented Unit Input Coefficient Matrix.* We get:

$$A^\star p(1 + \pi) = p \tag{6}$$

This is an eigenequation that (given the properties of $A^\star$) has a (i) unique and (ii) positive solution, p, for a positive rate of profit π. However, the price vector p is not *in general* proportional to λ, the vector of labor values.

The maximum eigenvalue of $A^\star$ is $0 < \frac{1}{1+\pi} < 1$.

> **Ricardo's Result:** If the proportion of *labor directly required* to *labor indirectly required* is the same in all lines of production, then the vector of prices, p, will be proportional to the vector, λ, of labor values.

Marx called this condition *equal organic composition of capital.*

Karl Marx's Puzzle: Why, in general, are there are profits at all? How could there be? Assuming:

(1) *Marketplace Conservation of Value:* equals are exchanged for equals in the marketplace; and

(2) *The Labor Theory of Value:* the natural price of a commodity is its labor value,

How could a capitalist obtain an output, by combining factor inputs, that could be exchanged on the market for more than the sum of the prices of its inputs?

The Price of Labor: Ricardo established that the *labor value* of a unit of labor itself is equal to the amount of labor directly and indirectly required for the production of the commodities required to produce (and reproduce) that unit of labor.

$$\lambda_0 = b\lambda \tag{7}$$

Marx's Main Point: The Capitalist could (under conditions of competition with goods that exchange in proportion to their labor values) earn a *profit* only if there existed a commodity whose *use value* had the property of being *a source of value* — i.e., whose consumption is itself "an embodiment of labor, and consequently, a creation of value." Is there such a commodity? Marx thought, "yes," it's *Labor Power.*

- **Labor Power.** The ability to labor. The worker sells his or her ability to labor for a period of time.
- **Labor-Value of Labor Power.** The labor-value of the ability to labor is the amount of laboring required directly and indirectly to produce those commodities (food, shelter, clothing, etc.) necessary for restoring that used up ability to labor.

If the economy is capable of producing Surplus Value, then *the labor value of a worker's labor power* must be strictly less than *the number of hours of labor extracted from the worker during that time period.*

In other words, in each line of production, the capitalist will complete a cycle of production with goods embodying more hours of labor than were embodied in the inputs used up to produce the output.

Marxian Exploitation: If the Capitalists earn a profit, then the Working Class is *exploited:* ***surplus labor value*** has been extracted from them in the form of hours of laboring over and above what is socially necessary to reproduce their ability to labor for another day.

Making Marx Precise. Let V_A be the labor value of commodity A (i.e., the quantity of labor directly and indirectly required for the production of a single unit of commodity A). Let V be the labor value of a single unit of *labor power.*

$$
\begin{aligned}
L_A + A_A V_A + B_A V_B + \cdots + N_A V_N &= O_A V_A \\
L_B + A_B V_A + B_B V_B + \cdots + N_B V_N &= O_B V_B \\
&\vdots \\
L_N + A_N V_A + B_N V_B + \cdots + N_N V_N &= O_N V_N
\end{aligned}
$$

Where $L = L_A + L_B + \cdots + L_N$.

The value of Labor Power is derived from the real wage with the following equation:

$$A_W V_A + B_W V_B + \cdots + N_W V_N = LV \tag{8}$$

Explanation of Notation:

- V Labor value of one unit of Labor Power.
- V_J Labor value of commodity J.
- L Total amount of labor employed in the economy.
- A Total amount of commodity A used up as input in the economy.
- O_J The amount of commodity J produced as output in the economy.

Summing up the system of equations, and equation (8), and simplify by substituting $A = A_A + A_B + \cdots + A_N + A_W$, etc., we get:

$$L + AV_A + \cdots + NV_N = O_A V_A + \cdots + O_N V_N + LV \tag{9}$$

$$L(1 - V) = V_A(O_A - A) + \cdots + V_N(O_N - N) \tag{10}$$

The term on the lefthand side — that is, $L(1 - V)$ — is the *surplus labor value* extracted from each unit of labor input multiplied by the total amount of labor employed in the economy as a whole. The term on the righthand side is labor value of the physical surplus produced in the system.

So, $\sum_{J \in \{A,B,\ldots,N\}} (O_J - J)$ is the amount of physical surplus produced in the given cycle of the economy.

And, $\sum_{J \in \{A,B,\ldots,N\}} V_J(O_J - J)$ is the labor value of the physical surplus produced.

Theorem of Surplus Labor Value: The surplus labor value extracted from each unit of labor input multiplied by the total number of units of labor employed in the economy *is equal to* the labor value of the physical surplus produced in the economy.

$$L(1 - V) = V_A(O_A - A) + \cdots + V_N(O_N - N)$$

If the labor value of the physical surplus is positive (i.e., if profit is generated), then $L(1 - V) > 0$. And so $1 > V$, which means that the amount of labor socially necessary to produce one unit of labor is less than the amount of value created by that one unit of labor.

The theorem derives from two assumptions: (1) **The Labor Theory of Value**, and (2) that Labor Power can be distinguished from the activity of laboring.

Upshot of the Theorem: the total number of hours of labor in a productive economy (i.e., one that generates profit) will be greater than the total number of labor hours socially necessary to produce that number of labor hours.

In other words, the Working Class will labor for more hours than would be necessary to sustain them. Because the Capitalists appropriate the Surplus Value in the form of profits, the Working Class is thereby exploited: they are creating more value than they are receiving.