Interpretations of Probability

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Frequency-type Interpretations of Probability

Frequentism identifies probabilities with *frequencies.* Very Roughly: the probability that *A is B* = the proportion of *A*s that are *B*s. There are several different ways to cash this idea out:

Example: The probability that this coin lands heads when tossed = the number of times the coin lands heads when tossed, divided by the total number of times that the coin is tossed.

1. ***Actual Finite Frequentism.*** The probability of an event = its actual relative frequency.

   FINITE FREQUENTISM: The probability of an attribute A in a finite reference class B is the relative frequency of actual occurrences of A within B.

   Example: I have an urn containing 100 marbles. Exactly 12 of the marbles are red. You reach in and grab a marble. What is the probability that the marble you've selected is red? *Answer:* There are 12 red marbles, and 100 marbles total; so $= \frac{12}{100}$.

   See Alan Hajek's paper "Mises Redux — Redux: Fifteen Arguments Against Finite Frequentism" *Erkenntnis.* 1997.

   Hajek's Fifteen Arguments Against Actual Finite Frequentism:

   (1) *The Reference Class Problem.* (2) *Typing Events may Change Probability.* (3) *Probabilities of Local Events can be Counterfactually Independent of Distant Events.* (4) *An Argument from Concern.* (5) *Actual Frequentism Commits one to a Surprisingly Rich Ontology.* (6) *Actual Frequentism = Operationalism about Probability.* (7) *Chance is Supposed to Explain Stable Relative Frequencies.* (8) *Attributes with No Occurrences have Undefined Relative Frequencies.* (9) *If B Occurs Once, A has Probability* 0 *or* 1 *: Local Determinism.* (10) *Universal Generalizations and Existential Statements.* (11) *Intermediate 'Probabilities' in an Deterministic World.* (12) *Finite Frequentism Generates Spurious Biases.* (13) *Finite Frequentism Generates Spurious Correlations.* (14) *All Irrational Probabilities, and Infinitely Many Rational Probabilities, 'Go Missing'.* (15) *Non-Frequentist Considerations Enter our Probabilistic Judgments: Symmetry, Derivation from Theory. . . .*

   Main Problem: It seems to get the wrong results. Suppose a fair coin is flipped once. It comes up heads. The coin is never flipped again. Is the probability of *heads* = 1, or $= \frac{1}{2}$?

2. ***Hypothetical Frequentism.*** The probability of an event = proportion of times it *would* occur out of an infinite number of trials.

   HYPOTHETICAL FREQUENTISM: The probability of an attribute A in a reference class $B = p$ if and only if the limit of the relative frequency of occurrences of A within B would be p if B were infinite.

   See Alan Hajek's paper "Fifteen Arguments Against Hypothetical Frequentism" *Erkenntnis.* 2009.

   Hajek's Fifteen Arguments Against Actual Finite Frequentism:

(1) *An Abandonment of Empiricism.* (2) *The Counterfactuals Appealed to are Utterly Bizarre.* (3) *There is no Fact of what the Hypothetical Sequences Look Like.* (4) *There Could be More than One Limiting Relative Frequency: The Problem of Ordering.* (5) *There Could be Less than One Limiting Relative Frequency: the Limit may not Exist.* (6) *The Limit may Equal the Wrong Value.* (7) *Hypothetical Frequentism's Order of Explanation is Back-to-Front.* (8) *The Limit Might Exist When it should Not.* (9) *Subsequences can be Found Converging to Whatever Value You Like.* (10) *Necessarily Single-Case Events.* (11) *Uncountably Many Events.* (12) *Exchangeability, and Independent, Identically Distributed Trials.* (13) *Limiting Relative Frequency Violates Countable Additivity* (14) *The Domain of Limiting Relative Frequencies is not a Field.* (15) *According to Hypothetical Frequentism, there are no Infinitesimal Probabilities.*

Major Problem: What counts as the "same trial"? Events that, intuitively, cannot be repeated have probabilities. Who cares about what happens in the long run?

3. ***Propensity.*** The probability of an event occurring = the *propensity* (or disposition, or tendency, etc.) of the world to produce events of that type. Objects have dispositional properties which ground facts about hypothetical frequencies.

Belief-type Interpretations of Probability

1. ***The Evidential/Logical Interpretation.*** We understand probability as a generalization of the notion of deductive entailment. Probability captures the idea that *A strongly supports B*, without *entailing* it.

2. ***The Personal Probability Interpretation.*** There are two types:

   - *Actual Personal Probabilities.*
   - *Ideally Rational Personal Probabilities.*

   The Bayesian Program:

   Stage 1: ***Precision.*** Individuals have real-valued degrees of belief.

   Stage 2: ***Synchronic Rationality (Probabilism).*** A rational agent's degrees of belief conform to the probability axioms.

   Stage 3. ***Diachronic Rationality (Conditionalization).*** The rational way to "update" on some information is to conditionalize on it.

   Probability is degree of belief.