The Allais Paradox & Risk-Aversion

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The Sure-Thing Principle

Another constraint underlying expected utility theory is the Sure-Thing Principle (or, in the vN-M framework, the "independence" axiom).

Sure-Thing Principle If f, g, and f^*, g^*, are such that

(i) for all $s \in \neg E$, $f(s) = g(s)$ and $f^*(s) = g^*(s)$,

(ii) for all $s \in E$, $f(s) = f^*(s)$ and $g(s) = g^*(s)$,

Then $f \succ g$ if and only if $f^* \succ g^*$.

SURE-THING PRINCIPLE

	E	$\neg E$
f	X	Z
g	Y	Z
f*	X	Z^*
g*	Y	Z^*

$f \succ g$ if and only if $f^* \succ g^*$

The principle is meant to formalize *sure-thing reasoning:* if two gambles agree on what happens if one event obtains, then your preferences between them should depend only on your preference between what happens if this event doesn't obtain.

The Allais Paradox. Maurice Allais presented a potential counterexample to the principle. Consider the following two lotteries: (L_1) an 11% chance of winning \$1,000,000; (L_2) a 10% chance of winning \$5,000,000. Which would you prefer?

Now consider two more lotteries: (L_3) a 100% chance of winning \$1,000,000; (L_4) a 10% chance of winning \$5,000,000 and an 89% chance of winning \$1,000,000. Which would you prefer?

Allais hypothesized that people would prefer L_2 to L_1 and would prefer L_3 to L_4. But these preferences violate the Sure-Thing Principle.

Notice, also, that there is no utility-function such that $U(L_2) > U(L_1)$ and $U(L_3) > U(L_4)$. Even if money has decreasing marginal utility, these preferences cannot be rationalized with expected utility theory.

THE ALLAIS PARADOX

	Tickets 1	2–11	12–100
L_1	\$1,000,000	\$1,000,000	\$0
L_2	\$0	\$5,000,000	\$0
L_3	\$1,000,000	\$1,000,000	\$1,000,000
L_4	\$0	\$5,000,000	\$1,000,000

If it's rational to prefer L_2 to L_1 and to prefer L_3 to L_4, then we have a counterexample to the Sure-Thing Principle.

Arguments for the Sure-Thing Principle

1. *Dominance.* Harsanyi defends the principle with the following argument:

   > [The Sure-Thing Principle] is essentially a restatement, in lottery-ticket language, of the *dominance principle* . . . The dominance principle says, If one strategy yields a better outcome than another does under *some* conditions, and never yields a worse outcome under *any* conditions, then always choose the first strategy, in preference over the second. On the other hand, the Sure-Thing Principle essentially says, If one lottery ticket yields a better outcome under *some* conditions than another does, and never yields a worse outcome under *any* conditions, then always choose the first lottery ticket. Surely, the two principles express the very same rationality criterion! (Harsanyi 1977, p. 384)

   Is this argument compelling?

2. *No Interaction Effects.* Samuelson defends a related principle in the following way:

   > Either heads *or* tails must come up: if one comes up, the other cannot; so there is no reason why the choice between [X] and [Y] should be 'contaminated' by the choice between [Z] and [Z^*]. (Samuelson 1952, p. 672-3)

   How is this argument supposed to go? Does it work?

The Redescription Strategy

The Allais Paradox is only a problem for the Sure-Thing Principle (and expected utility theory) if we've correctly specified the outcomes of the lotteries. But have we?

Broome argues that no rational agent can violate the Sure-Thing Principle — that any intuitive counterexample to the principle is not really a counterexample after all.

> All the [rationalizations of the Allais preferences] work in the same way. They make a distinction between outcomes that are given the same label in [the initial presentation], and treat them as different outcomes that it is rational to have a preference between. And what is the argument that Allais' preferences are inconsistent with the Sure-Thing Principle? It is that all the outcomes given the same label [initially] are in fact the same outcome. If they are not . . . [the decision-problem] will have nothing to do with the Sure-Thing Principle. Plainly, therefore, the case against the Sure-Thing Principle is absurd. It depends on making a distinction on the one hand and denying it on the other. (Broome 107)

What's Broome's point here? Is he right?

THE ALLAIS PARADOX (REDESCRIBED)

	Tickets		
	1	2–11	12–100
L_1	\$1,000,000	\$1,000,000	\$0
L_2	\$0	\$5,000,000	\$0
L_3	\$1,000,000	\$1,000,000	\$1,000,000
L_4	***Regret***	\$5,000,000	\$1,000,000